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Two stream instabilities in semiconductors exhibiting negative differential mobility

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The two stream instability in a semiconductor exhibiting negative differential mobility is considered. We assume that the transferred-electron mechanism creates the two streams in the semiconductor. For a stationary electron in the upper stream, there exists a wave which moves with the phase velocity equal to the drift velocity of the electrons in the lower stream. This wave grows for the negative values of the differential mobility. The condition for the growing convective instability is derived under different assumptions. We have shown that the backward wave solution which is generally heavily damped can lead to the convective instability in the two valley semiconductor.

1. INTRODUCTION

The instabilities in semiconductor plasmas have been studied by various authors (Stelle *et al* 1969, Vural *et al* 1968, Willardson *et al* 1966, Glicksman 1965). Gueret (1971) has studied the convective and absolute instabilities in a semiconductor exhibiting the negative differential mobility. He pointed out that the forward wave solution shows the convective instability. The backward wave solution was found to be diffusion dependent and heavily damped. For this solution he pointed out the possibility of the existence of absolute instability. We have considered the similar problem for a two valley semiconductor. We have derived the conditions for the existence of convective instabilities under different possible assumptions and have shown that in the case of two valley semiconductor the backward wave solution can be convectively unstable, under certain approximations and assumptions.

2 THEORY AND DISCUSSION

We assume that the transferred-electron mechanism (Ridley *et al* 1961, Hilsum 1962) creates the two streams, with n_1 electrons per unit volume in the lower valley of the conduction band and n_2 in the higher energy valley such that n_1+n_2 remains constant in a uniform, infinite semiconductor in which the carriers are drifting with velocities v_1 and v_2 ($v_1 >> v_2$) respectively in the lower and upper valley of the conduction band under the influence of an applied field E_0'

For a small wavelike perturbation of the form $\exp[j(wt-kz)]$, the dispersion relation for the electrons in the lower valley of the conduction band is obtained by solving the linearized set of equations :

$$j_{z1} = w_{c1}\epsilon E_{z1} + V_{01}\rho_1 - D_1 \frac{\partial \rho_1}{\partial z} \quad \dots (1)$$

$$\epsilon \frac{\partial E_{z1}}{\partial z} = \rho_1 \quad \dots (2)$$

$$\nabla \times H = j_{z1} + jw\epsilon E_{z1} = 0, \quad \dots (3)$$

where D_1 is the diffusion constant and $w_{c1} = \mu_1 n_1 e/\epsilon$ the dielectric relaxation frequency, μ_1 being the differential mobility $\partial v_1/\partial E_0$ E_0 is the applied electric field, ϵ the dielectirc constant of the medium and w the frequency of the wave As we have already stated that these equation are written in the small signal approximation and the subscript 1 here simply means the corresponding quantities for the lower valley electrons

In writing the above equations, we have assumed that the carriers are drifting in the direction of the applied electric field, say the z axis After combining eqs (1) to (3), one obtains the following dispersion relation

$$1 - j\frac{w_{c1}}{(w - kv_1 - jk^2 D_1)} = 0, \quad \dots (4)$$

which for the two valley semiconductor reduces to

$$1 - j\sum_{i=1,2} \frac{w_{ci}}{(w - kv_i - jk^2 D_i)} = 0. \quad \dots (5)$$

where $i = 1$ for the lower valley electrons and $i = 2$ for higher valley electrons We analyse the dispersion relation (5) under the various simplified assumptions

(1) If the drift velocity v_2 of the electrons in the upper valley approaches to zero then in absence of diffusion ($D_1 = D_2 = 0$) the dispersion relation (5) simplifies to

$$k = w/v_1 - jw_{c1}/v_1 \quad \dots (6)$$

The solution (6) describes a forward wave which moves with the phase velocity equal to the drift velocity v_1 of the electrons in the lower valley The growth or decay of this wave depends upon the sign of w_{c1} and hence of μ_1 For $\mu_1 < 0$ the wave will grow and for $\mu_1 > 0$, the wave will decay.

(2) Under the approximation when $v_2 = 0$ and $D_2 = 0$, the equation (5) simplifies to

$$k = j\frac{v_1}{2D_1} \pm j\frac{v_1}{2D_1}\left[1+4\frac{w_{c1}D_1}{v_1^2}+4j\frac{wD_1}{v_1^2}\right]^{\frac{1}{2}} \qquad \ldots \quad (7)$$

This is the same expression as derived by Gueret. In the limit of small diffusion and low frequencies ($w_{c1}D_1/v$ and $wD_1/v << 1$), eq (7) further simplifies and gives the following two roots for k

$$k_1 = w/v_1 - jw_{c1}/v_1$$

which is similar to eq (6) and is independent of diffusion, and

$$k_2 = -w/v_1 + jv_1/D_1 \qquad \ldots \quad (8)$$

This is the backward wave solution which is heavily damped being diffusion dominated.

(3) When $D_1 = D_2 = 0$ but $v_2 \neq 0$, eq (5) now gives the following two complex roots for k

$$k\pm = \frac{1}{2v_1 v_2}\left[(b \pm c) + j(\pm d - a)\right], \qquad \ldots \quad (9)$$

where

$$a = w_{c1}v_2 + w_{c2}v_1, \quad b = (v_1 + v_2)w,$$

$$c = \frac{1}{\sqrt{2}}\left[P + (P^2 + Q^2)^{\frac{1}{2}}\right]^{\frac{1}{2}},$$

$$d = \frac{1}{\sqrt{2}}\left[-P + (P^2 + Q^2)^{\frac{1}{2}}\right]^{\frac{1}{2}},$$

$$P = b^2 - a^2, \quad Q = W[v_1v_2(w_{c1} + w_{c2} + w_{c2}v_1^2 + w_{c1}v_2^2]$$

Out of the two solutions represented by eq (9) the one with upper sign corresponds to the forward wave and the other solution corresponds to the backward wave. The gain or loss per unit length, k_{Im}, ($k = k_{Re} + jk_{Im}$) for the forward wave is given by

$$k_{Im} = (d - a)/2v_1v_2 \qquad \ldots \quad (10)$$

The condition for the growing convective instability is

$$k_{Im} > 0 \qquad \ldots \quad (11)$$

for $k_{Re} > 0$ otherwise for $k_{Re} < 0$ the condition (11) gives the decaying waves solutions. To satisfy eq (11), (d-a) should be positive. Since d is always a positive quantity and therefore the sufficient condition for the existance of the growing convective instability is that a should be negative i.e.,

$$\frac{w_{c1}}{w_{c2}} + \frac{v_1}{v_2} < 0, \qquad \ldots \quad (12)$$

Since v_1/v_2 is greater than one and therefore to satisfy the eq (12) we must have

$$\frac{w_{c1}}{w_{c2}} > -1, \qquad \dots (13)$$

i e the ratio of the dielectric relaxation frequencies and hence the differential mobilities in the two streams must be greater than minus one. If a vanishes $\left[-\frac{w_{c1}}{w_{c2}} = \frac{v_1}{v_2}\right]$, then Q also vanishes and consequently d vanishes In other words k_{Im} becomes zero and the system gives the satable solutions

(4) When the phase velocity of the electrons in the upper stream is greater than the drift velocity of the electrons in the stream, then for $D_2 = 0$, the following two complex roots of k are obtained

$$k_{\pm} = \frac{v_1}{2D_1}\,[1 \pm \alpha \mp j\beta], \qquad \dots (14)$$

where

$$\alpha = \frac{1}{\sqrt{2}}\,[a_1 + (a_1^2 + a_2^2)^{\frac{1}{2}}]^{\frac{1}{2}},$$

$$\beta = \frac{1}{\sqrt{2}}\,[-a_1 + (a_1^2 + a_2^2)^{\frac{1}{2}}]^{\frac{1}{2}},$$

$$a_1 = 1 - \frac{4}{w_{D1}} \cdot \frac{W_{c1}}{1+(w_{c2}/w)^2},$$

$$a_2 = \frac{\left[\frac{4}{w_{D1}}\, w\left(1 + \frac{w_{c1}\, w_{c2}}{w^2}\right)\right]}{1+(w_{c2}/w)^2}$$

and $w_{D1} = v_1^2/D_1$ the diffusion frequency

It can be noted that for the backward wave the condition for the connective instability is

$$\beta > 0 \qquad \dots (15)$$

which means that a_1 should be negative In other words $|w_{c1}| > (v_1^2/4D_1)$ $[1+(w_{c2}/w)^2]$ gives the connective instability for the backward wave solution Here it appears that for the forward wave there can not exist the convective instability because that will require that $\beta < 0$ which is not possible

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